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Hedging and Portfolio Optimization of Indian Index Futures Contracts using Multivariate GARCH Models

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ABSTRACT: This research focuses on optimizing a hedged portfolio of stock index futures using a fuzzy multiobjective linear programming (FMOLP) technique. FMOLP transforms a multiobjective optimization problem into a single objective with the help of membership functions. This technique allows choices in solution. In this research, multivariate GARCH models like SGARCH, EGARCH and, GJR-GARCH are applied to perform hedging decisions. The parameters used to perform hedging decisions are hedge ratio and hedging effectiveness. Data of NIFTY50, BANKNIFTY, and NIFTYIT index future is downloaded from nseindia.com from 1 Jan 2006-31 Dec 2015. The equally weighted or 1/N-portfolio approach is adopted to construct hedged portfolios of the indices. The parameters taken for optimizing hedged portfolios are return, risk, Sharpe ratio, Treynor ratio, and coefficient of variation (CV). The measure of risk used to perform optimization is the Conditional Value at Risk (CVaR). As the stock market is always faced with the problem of increased volatility and unexpected price fluctuations, hedging strategies sprung up in recent years to solve this problem. Investment companies and corporations, fund managers apply hedging strategies to reduce their exposure to various risks. This study will help investors in selecting a better hedging strategy for a longer time horizon to protect their portfolio from potential losses.

Keywords: CVaR, FMOLP, hedge ratio, hedging effectiveness, index futures, portfolio optimization.

Abbreviations: GARCH, generalized autoregressive conditional heteroscedasticity; SGARCH, simplified GARCH; EGARCH, exponential GARCH; GJR-GARCH, glosten-jagannathan-runkle GARCH; FMOLP, fuzzy multiobjective linear programming; CV, coefficient of variation; CVaR, conditional value at risk.

I. INTRODUCTION

Portfolio optimization occupies an important place in investment. It enables an investor to intellectually select different assets in a portfolio to maximize the return and minimize the risk. Prof. H.M. Markowitz is the father of modern portfolio theory who propounded the famous mean-variance theory of portfolio optimization [1-2]. It is based on maximizing the expected return at a given level of market risk or minimizing risk at a given level of return. In this theory, the risk is measured by the variance of returns [1]. But this theory is criticized due to its quadratic nature which makes it a complicated problem to solve. Since then various measures of risk have been introduced in the literature like semi-variance [3], mean absolute deviation [4], semi absolute deviation [5] etc. Recently quantile based risk measures have become more popular as they determine the portfolio losses occurring in the tail of loss distribution. A well-known quantile-based risk measure is the Value-at-Risk (VaR). It is defined as the worst expected loss at a target horizon, according to a determined confidence level [6]. But it is not a consistent risk measure as it only provides a lower bound for losses, without distinguishing between situations in which losses may be slightly or much higher than the threshold [5]. Due to these difficulties, an alternative measure of risk called Conditional Value at Risk (CVaR) is introduced by Rockafellar and Uryasev in 2000 [7]. It is defined as the weighted average of the VaR and the losses strictly exceeding the VaR [7-9]. CVaR is a convex function, and its minimization model can be condensed into a simple linear programming formula, making it a widely used and studied area of research and development.

In this research, optimization of the selected portfolio is performed by minimizing Conditional Value at Risk (CVaR). As parameters of the financial market are uncertain, they can be effectively modelled using fuzzy numbers. In this research, FMOLP technique is adopted to optimize a hedged portfolio of index futures. This technique reduces the multiobjective problem into a single objective with the help of membership functions [10-12]. In this research, a multiobjective problem with five objectives viz. return, risk, Sharpe ratio, Treynor ratio and coefficient of variation (CV) is taken into account. A hedged portfolio of NIFTY50, BANKNIFTY and NIFTYIT index futures is formed using multivariate GARCH models like SGARCH, EGARCH and GJR-GARCH. Daily spot and future price of these indices are downloaded from nseindia.com from January 2006-December 2015. The reason behind taking such a large data is that it covers all the important properties of time series like stationarity, ARCH effects, skewness, kurtosis etc. This research will provide a substantial contribution to the literature in the sense that before optimization, hedging is done to reduce the unexpected fluctuations in the market borne by an investor. Hedging is done based on the portfolio approach or the mean-variance approach. Secondly GARCH models used in this research has not been applied so far in the literature for hedging Indian index futures contract. Different criteria like Sharpe ratio, Treynor ratio, and CV have not been used before in the optimization of index futures. A higher Sharpe and Treynor ratio helps in selecting a better portfolio with higher return and minimum risk. Similarly, a lower CV enables an investor to select a portfolio with minimum risk. FMOLP is also used for the first time in the optimization of a hedged portfolio of Indian index futures contracts.

The motivation for this research comes from the fact that the proposed models in this study have never been considered in the Indian stock market context. GARCH techniques are able to model volatility of financial returns [13]. The models are so chosen that they are easy to compute. They can be easily extended to include complex dynamics of stock market. Also the models involve less number of parameters, so they are free from the problem of convergence. A hedged portfolio of Indian index futures contract is optimized which came out to be a new area of research on which not much consideration has been paid in recent years. FMOLP technique is also applied for the first time in optimizing a hedged portfolio of Indian index future contracts. The parameters adopted for optimization consists of return, risk, Sharpe ratio, Treynor ratio, and CV. FMOLP together with these parameters has never been considered for optimizing a multiobjective optimization problem.

As far as our knowledge, there exist very few studies involving optimization of hedged portfolios using FMOLP in Indian stock market context. This research focuses on multiobjective portfolio optimization of Indian index futures contract using FMOLP. Along with return and risk, other criteria employed for optimization includes Sharpe ratio, Treynor ratio and coefficient of variation. These factors are known to improve the performance of an asset to a great extent. The multiobjective optimization model in this study aims at maximizing returns, Sharpe and Treynor ratio and minimizing risk (CVaR), and coefficient of variation. Optimization is done under some realistic constraints like upper and lower bounds for invested capital, no short selling, and full utilization of the invested capital. Equally weighted portfolio or 1/N-portfolio approach is used to construct hedged portfolios of Indian index futures contract. The resultant optimization problem is solved using FMOLP.

The approach suggested in this research offers some new features and a much simpler framework to solve a multiobjective optimization problem. On observing the existing literature on portfolio selection, the present study considers a hedged portfolio of Indian index futures contract. This approach has not been considered so far in the context of the Indian stock market. This study considers integrated framework which not only performs hedging decisions using multivariate GARCH models but also helps to evaluate the performance of assets via maximizing return, Sharpe ratio, Treynor ratio and minimizing risk, and CV. The multivariate GARCH models in this study have less number of parameters which removes the problem of convergence and become easier to evaluate. Most of the optimization problems are non-linear. It is very difficult to solve them. They should be made linear to apply optimization techniques. The proposed multiobjective optimization model with all the objectives in this study are linear which is much easier to solve. The equally weighted or 1/N-portfolio approach is adopted for hedging. The advantage of this is they naturally take a value-approach preferred by many investors. They are highly diversified and their long term performance appears to be superior owing to large sample sizes. The decision making done in this research is compared with the decision making done by Singh (2017) and found that our approach is providing much better results with maximum hedging effectiveness [22].

This paper is categorized as follows: next section covers the literature review. Section III includes research methodology. Section IV presents numerical illustration with data description and results and discussion while Section V concludes the paper.

II. LITERATURE REVIEW

Njegić *al.*, (2019) observed the effect of structural breaks on optimal weights, hedge ratios and hedging effectiveness of portfolios. DCC-EGARCH model with and without structural breaks is applied for hedging [14]. Gallien *et al.*, (2018) have applied mean variance optimization technique in hedging and portfolio optimization with transaction cost over a portfolio of a risk- free bond, a futures contract and an asset [15]. Sarwar *et al.,* (2019) investigated the spillover effect in volatility between stock market returns and crude oil returns. BEKK-GARCH, DCC-GARCH, cDCC-GARCH and GO-GARCH are applied to find optimal portfolio weights and hedge ratios [16]. Chakravorty and Awasthi (2018) proposed a conservative, global tactical asset allocation strategy for a hypothetical, European investor and highlight the benefits of dynamic currency hedging over static hedging [17]. Davari-Ardakani *et al.,* (2015) proposed a multi-period portfolio optimization model that uses hedging decisions in a dynamic setting [18]. Luo *et al.,* (2015) applied the mean-variance portfolio optimization technique to a fund of hedge funds and solve it by Lagrange's multiplier method. Portfolios are formed with the help of the models OGARCH, Markov switching and EWMA model [19]. Syriopoulos *et al.,* (2015) studied the time-varying risk–return properties of the BRICS capital markets. Also models potential time-varying correlations and volatility spillover effects with the US stock market are investigated. VAR (1)–GARCH (1, 1) model is applied in computing effective portfolio hedge ratios optimal portfolio weights for diversified asset allocation [20]. Christopher applied mean-variance optimization to a currency-hedged portfolio. Optimization is carried out relative to a benchmark portfolio consisting of real assets [21]. Liu *et al.,* applied a rollover hedge strategy for the long-term exposure of a well-diversified portfolio. Dynamic programming is used to obtain the optimal proportion of stock index futures contracts [22]. Li (2009) applied Mean-CVaR model in optimizing and hedging a portfolio under the conditions with and without expected return requirement [8].

From the literature, it is concluded that the majority of the researches use mean-variance portfolio optimization model to optimize a portfolio of options, stocks and bonds. Most of the investigations in this regard are related to the developed nations. There are very few studies in the Indian stock market framework. GARCH models used in this research have not been used so far in the literature. An attempt is made to use these models in hedging decisions and optimize hedged portfolios with the help of FMOLP.

III. METHODOLOGY

This research is performed in the following steps:

A. Hedging

The first step to conduct this research is to perform hedging decisions with proposed multivariate GARCH models. Hedging is an advanced investment strategy to decrease or transfer the risk of spot portfolio without buying insurance policies. As in risk/return trade-off, along with reducing risk, hedging results in lower returns. Hedging is done to minimize the losses, not to save cost or earn profits [23]. The two crucial inputs of hedging are hedge ratio and hedging effectiveness.

Hedge ratio. To hedge with the futures contract, hedge ratio plays an important part. It helps an investor to determinehow many future contracts are needed to minimize the risk of spot market.The value of the hedge ratio which minimizes the variance of the hedged portfolio is called the Minimum Variance Hedge Ratio (MVHR) [23]. The value of the hedge ratio should be as minimum as possible because higher hedge ratios require higher investment [24]. If $R_{s,t}$ and $R_{f,t}$ are the returns of the spot and future portfolio at any time t based on the information upto time t-1, then the minimum variance hedge ratiopis given by [25, 26].

$$\rho = \frac{\operatorname{cov}(R_{s,t}, R_{f,t})}{\operatorname{var}(R_{f,t})} \tag{1}$$

Hedging effectiveness. This technique is used to analyze the effectiveness of a hedging strategy. It is expressed in percentage and determine how efficient a hedging instrument is to protect from potential losses**.** It is the extent to which hedging an instrument actually reduces risk [27]. Given the variance of the hedged and the unhedged return, hedging effectiveness is given by

$$\mathrm{HE} = \frac{\mathrm{Var(U)} - \mathrm{Var(H)}}{\mathrm{Var(U)}} \tag{2}$$

where Var (U) = variance of unhedged or spot portfolio

Var (H) = variance of hedged portfolio given by

$$\mathrm{Var(H)} = \mathrm{Var}(R_{s,t}) + \rho^2 \mathrm{Var}(R_{f,t}) - 2\rho \operatorname{cov}(R_{s,t}, R_{f,t}) \tag{3}$$

$\mathrm{Var}(R_{s,t})$ =Variance of spot portfolio

$\mathrm{Var}(R_{f,t})$ =Variance of future portfolio

ρ =Hedge ratio given by (1)

$\operatorname{cov}(R_{s,t}, R_{f,t})$ = Covariance between spot and future portfolio estimated using GARCH models.

B. Models used in hedging

The dynamic hedge ratios used in this research are estimated using the multivariate GARCH models: SGARCH, EGARCH, and GJR GARCH. These models are applied to the return series of spot and futures price of the indices taken.

Model 1: SGARCH model. This model is given by Harris *et al.*, (2007) [28]. It makes use of univariate GARCH models to evaluate the conditional variances for each individual return series and the series formed by the sum and difference of each pair of series. The covariance between the return series is then evaluated from these variance estimates. The model for spot returns based on the GARCH (1, 1) model is:

$$\left.\begin{aligned} R_{s,t} &= \mu_s + \varepsilon_{s,t}, \varepsilon_{s,t} = \sigma_{s,t} z_t \\ \sigma^2_{s,t} &= a + b\sigma^2_{s,t-1} + c\epsilon^2_{s,t-1} \end{aligned}\right\} \tag{4}$$

where μ_s is the conditional mean return, $\varepsilon_{s,t}$ is the residual term i.i.d. with zero mean and unit variance, a is the constant or the intercept term, b is the coefficient of GARCH term, it is the forecasted volatility from the past period, c is the coefficient of ARCH term, $\sigma^2_{s,t}$ is the conditional volatility of $r_{s,t}$and is set on the information known at the time t-1. The covariance-stationary condition for GARCH (1, 1) process is $a + b<1$. $a + b$ gives the level of persistence.

Similarly the model for future return is

$$\left.\begin{aligned} R_{f,t} &= \mu_f + \varepsilon_{f,t}, \varepsilon_{f,t} = \sigma_{f,t} z_t \\ \sigma^2_{f,t} &= a + b\sigma^2_{f,t-1} + c\epsilon^2_{f,t-1} \end{aligned}\right\} \tag{5}$$

Using the GARCH (1,1) model $\sigma^2_{s,t}$ and $\sigma^2_{f,t}$ can be found out. After that two series $R_{+,t} = R_{s,t} + R_{f,t}$ and $R_{-,t} = R_{s,t} - R_{f,t}$ are created and GARCH (1, 1) is used to find the conditional variance of these two series similar to model 4 and 5. And then the covariance between spot and future return can be calculated as

$$\sigma_{sf,t} = \frac{1}{4}(\sigma^2_{+,t} - \sigma^2_{-,t}) \tag{6}$$

The minimum variance hedge ratio is then obtained as

$$\rho = \frac{\operatorname{cov}(R_{s,t}, R_{f,t})}{\operatorname{var}(r_{f,t})} = \frac{\sigma_{sf,t}}{\sigma^2_{f,t}} \tag{7}$$

The measure of hedging effectiveness is then given by Eqn. (2)

Model 2: GJR-GARCH model. To model the 'leverage effect' present in the return series Glosten, Jagannathan, and Runkle (1993) proposed this model [29]. According to this model, a dummy variable is included in the original GARCH model to capture the asymmetric effect in the return series. This model also captures positive and negative innovations to returns which laid different impacts on conditional volatility. GJR-GARCH model is represented by the expression:

$$\begin{aligned} \sigma^2_{s,t} &= a + b\sigma^2_{s,t-1} + c\epsilon^2_{s,t-1} + \epsilon^2_{s,t-1} I_{t-1} \\ \sigma^2_{f,t} &= a + b\sigma^2_{f,t-1} + c\epsilon^2_{f,t-1} + \epsilon^2_{f,t-1} I_{t-1} \\ \sigma_{sf,t} &= a + b\sigma_{sf,t-1} + c(\epsilon_{s,t-1}\epsilon_{f,t-1}) \end{aligned} \tag{8}$$

where I_{t-1} is a dummy variable and I_{t-1}=1 if ϵ_{t-1}<0 (for both spot and future returns); otherwise I_{t-1}= 0. This will capture the negative returns more precisely and will help in making correct hedging decisions. The minimum variance hedge ratio is then obtained by (7) and the corresponding hedging effectiveness by (2).

Model 3: EGARCH model. This model was proposed by Nelson (1991). It describes the asymmetric relationship between conditional mean and conditional volatility [30]. In this model, the conditional volatility may be expressed as follows:

$$\left.\begin{aligned} \epsilon_t &= \sigma_t z_t \\ \ln(\sigma^2_t) &= a + b\ln(\sigma^2_{t-1}) + c\left(\left|\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right| - \frac{\sqrt{2}}{\pi}\right) + d\left(\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right) \end{aligned}\right\} \tag{9}$$

In the above equation, conditional variance is an exponential function of the variables under study. This ensures its positive nature. The exponential character of this model shows that external unexpected shocks laid stronger influence on the predicted volatility. The value d>0 of the variable d indicates asymmetric effect. The negative value of d shows the existence of a 'leverage effect'.

C. FMOLP

The optimization of hedged portfolios formed using multivariate GARCH models is done using FMOLP. Zimmermann (1978) introduced a fuzzy linear programming problem in fuzzy environment. It is a linear programming optimization technique which converts a multiobjective problem into a single objective with the help of membership function [12]. The various steps involved in solving this optimization problem are as follows:

Portfolio selection problem. The notations used in the multiobjective portfolio selection problem with five objectives return, risk, Sharpe ratio, Treynor ratio and coefficient of variation are as follows:

$R_{j,t}$ =Return of the hedged portfolio of jth index future at time t

x_j =Proportion of the investment allocated in jth index future

y_j = Binary variable expressing whether a particular index future is included in the portfolio or not

$\text{i.e.}, yj = \begin{cases} 1, \text{if jth index future is included in the portfolio} \\ 0, \text{ otherwise} \end{cases}$

m = Number of stocks held in the portfolio (3)

φ_j = Risk of the jth index future obtained by CVaR

c_j =Coefficient of variation of the jth index future

s_j = Sharpe rartio of the jth index future

w_j = Treynor ratio of the jth index future

u_j = Upper bound for the jth index future

l_j = Lower bound for the jth index future

n= Total number of index future under study (3)

T=Total number of time periods considered.

Parameters used. The following parameters are used for optimization in this research.

1. Return: It is the profit earned on an investment. Generally it is calculated as

$$R_t = \frac{p_t - p_{t-1}}{p_{t-1}} \quad (10)$$

or logarithmic return

$$R_t = \ln \frac{p_t}{p_{t-1}} \quad (11)$$

In this research the return of the jth index future at time *t* is given by

$$R_{j,t} = \ln \frac{p_{j,t}}{p_{j,t-1}} \quad (12)$$

where $p_{j,t}$ is the price of the j^{th} index future at time *t*

Return of the hedged portfolio is given by

$$R_h = R_{s,t} - \rho R_{f,t} \quad (13)$$

where $r_{s,t}$ is the spot portfolio return, $r_{f,t}$ is the future portfolio return calculated using (12) and ρ is the hedge ratio given by (7). The return of the portfolio is given by

$$f_1(x) = \frac{1}{T} \sum R_{j,t} x_j \quad (14)$$

2. Risk: It is the loss incurred on an investment. In this research, it is given by CVaR [6] or the expected shortfall. It is a statistical technique that measures the amount of risk found in the tail of an investment portfolio. It can be obtained by taking a weighted average of the "extreme" losses in the tail of the distribution of possible returns, beyond the value at risk(VaR) cut-off point.

Let f(x,y) be the loss function where x is the decision vector belongs to a portfolio and satisfies the condition of short selling and expected return and y is a random vector representing the uncertainty of future returns. Let p(y) be the probability distribution of y. Let Ψ(x, α) be the probability that f(x,y) does not exceed the threshold α i.e.,

$$\Psi(x, \alpha) = \int_{f(x,y) \le \alpha} p(y) dy \quad (15)$$

If α(x) be the VaR for the loss random variable associated with x and any specified probability level β in (0, 1) i.e., if

$$\alpha(x) = \min\{\alpha \epsilon R : \Psi(x, \alpha) \ge \beta\} \quad (16)$$

Then CVaR is defined as

$$\varphi(x) = \frac{1}{1-\beta} \int_{f(x,y) \ge \alpha(x)} f(x,y) p(y) dy \quad (17)$$

Consequently risk of the portfolio becomes

$$f_2(x) = \sum \varphi_{j,t}(x) x_j \quad j=1\ldots.n, \; t=1\ldots T \quad (18)$$

Sharpe ratio: This ratio was given by William F. Sharpe and is known as the reward-to-volatility ratio. With the help of this ratio, one can know the expected return rate of an investment in comparison to its risk. Generally, higher value of Sharpe ratio shows higher risk-adjusted return [31]. It is calculated as

$$s = \frac{R_p - R_f}{\sigma_p} \quad (19)$$

where R_p = return of the portfolio.

R_f = risk free rate of return.

σ_p= standard deviation of portfolio's excess return.

For a portfolio, it is given by

$$f_3(x) = \sum s_j x_j \quad j=1...n \quad (20)$$

where s_j = (return of the portfolio- risk free return)/standard deviation of the portfolio

Treynor ratio: This ratio is given by Jack Treynor and is also a reward-to-volatility ratio. It is a metric for determining how much excess return was generated for each unit of risk taken on by a portfolio [32]. For a portfolio, it is given by

$$f_4(x) = \sum w_j x_j \quad j=1,\ldots, n \quad (21)$$

where $w_j = \frac{\text{return of the portfolio} - \text{risk free return}}{\text{beta of the portfolioi}}$

Coefficient of variation: It helps to determine the amount of volatility of an investment in comparison to the expected return. Coefficient of variation shows the risk per unit return. It is given by

$$f_5(x) = \sum c_j x_j \quad j=1,\ldots,n \quad (22)$$

where $c_j = \frac{\text{standard deviation of the jth index future}}{\text{return of the jth index future}}$

Constraints of the problem.

1. Sum of the proportions invested in each index future j should be equal to 1

$\sum x_j = 1$ j=1,…, n(budget constraint) (23)

2. Number of assets held in a portfolio is given by

$\sum y_j = m$ j=1,...,n (24)

3. Maximum proportion of the amount invested in each asset j

$x_j \le u_j \, y_j$ j=1,…,n (upper bound constraint) (25)

4. Minimum proportion of the amount invested in each asset j

$x_j \ge l_j \, y_j$ j=1…n (lower bound constraint) (26)

Upper and lower bounds are chosen to ensure a greater portfolio diversification.

Decision problem

$$\text{Max } f_1(x) = \frac{1}{T} \sum R_{j,t} x_j \quad j=1\ldots n, \; t=1\ldots T \quad (27)$$

$$\text{Min } f_2(x) = \sum \varphi_{j,t}(x) x_j \quad j=1\ldots n, \; t=1\ldots T \quad (28)$$

$$\text{Max } f_3(x) = \sum s_j x_j \quad j=1\ldots n, \; t=1\ldots T \quad (29)$$

$$\text{Max } f_4(x) = \sum w_j x_j \quad j=1\ldots n, \; t=1\ldots T \quad (30)$$

Min $f_5(x) = \sum c_j x_j \quad$ j=1…n, t=1…T (31)

Subject to

$$\sum x_j = 1 \tag{32}$$
$$\sum y_i = m \tag{33}$$
$$l_j \le x_j \le u_j \tag{34}$$
$$x_j \ge 0, j = 1 \ldots n, \text{(Short selling is not allowed)} \tag{35}$$
$$y_j \in \{0,1\} \tag{36}$$

On solving objective function (27) using the constraints (32) to (36) a solution X1 (value of objective function (27)) is obtained. Similarly X2 is obtained on solving (28) with constraints (32) to (36) and similarly X3, X4 and X5 are obtained on solving objective function (29), (30) and (31). From these solutions, select the highest and lowest value of all objective functions to be called as best upper bound (ub) and the worst lower bound (lb) respectively.

The membership function for the objective functions 27-31 taken are defined as:

$$\mu_{f_i(x)} = \begin{cases} 1 & ,\text{if } f_i(x) \ge ub \\ \frac{f_i(x)-lb}{ub-lb} & ,lb \le f_i(x) \le ub \\ 0 & ,\text{if } f_i(x) \le lb \end{cases} \quad \text{for } i=1, 3, 4$$

(maximization case) (37)

and

$$\mu_{f_i(x)} = \begin{cases} 0 & ,\text{if } f_i(x) \ge ub \\ \frac{ub-f_i(x)}{ub-lb} & ,lb \le f_i(x) \le ub \\ 1 & ,\text{if } f_i(x) \le lb \end{cases} \quad \text{for } i=2, 5 \text{ (minimization}$$

case) (38)

where $\mu_{f_i(x)}$ = satisfaction degree of the objective function for a given solution X

Now using equally weighted approach or 1/N- portfolio weighting approach, the multiobjective problem is converted to a single objective

$\text{Max} = 0.4515q_1 + 0.0515q_2 + 0.1645q_3 + 0.1765q_4 + 0.0645q_5$ Subject to

$f_i(x) - (ub - lb)q_i \ge lb$ i=1, 3, 4

$f_i(x) + (ub - lb)q_i \le ub$ i=2, 5

$0 \le q_i \le 1$ i= 1, 2, 3, 4, 5 (39)

$\sum x_j = 1$

$\sum y_j = m$

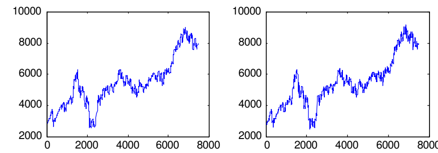
$l_j \le x_j \le u_j$

$x_j \ge 0$, j= 1…n

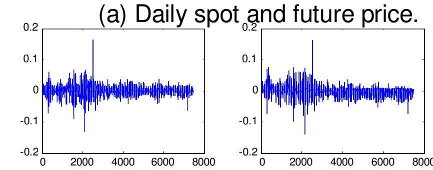
The solution to the above problem gives the first iteration. For the second iteration, replace old lower bound with the first iteration. Continue this process until an improved result is obtained or the investors are satisfied with the solution.

IV. NUMERICAL ILLUSTRATION

Data analysis and discussion. The dataset used in this research consists of a sample of daily spot and future price from three Indian index future contracts of NSE. Daily spot and future price of the indices CNX NIFTY50, BANKNIFTY and NIFTYIT are downloaded from nseindia.com for the period 1 Jan 2006 to 31 Dec 2015. A total of 7441 observations for NIFTY50, 6418 observations for BANKNIFTY and 3910 observations for NIFTYIT index are used. All futures price indices are continuous series and show some trends and fluctuations (Fig. 1(a), 2(a), 3(a)).

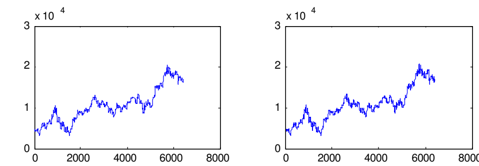


(a) Daily spot and future price.

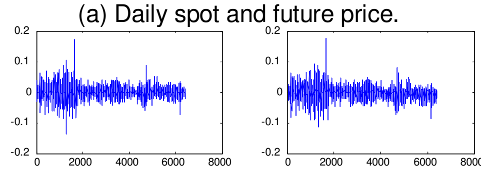


(b) Daily spot and future returns.

Fig. 1. Daily spot and future price series and return series of NIFTY50.

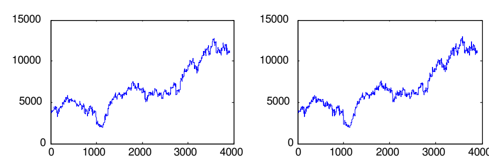


(a) Daily spot and future price.

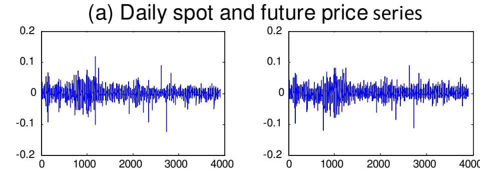


(b) Daily spot and future returns.

Fig. 2. Daily spot and future price series and return series of BANKNIFTY.



(a) Daily spot and future price series



(b) Daily spot and future returns.

Fig. 3. Daily spot and future price series and return series of NIFTYIT.

Hedge ratio and hedging effectiveness are estimated statistically, it is necessary to consider the statistical properties of a time series data under study. To test whether the two series are stationary or not, ADF (Augmented Dickey Fuller) test is conducted in EVIEWS software (Table 1-3). ADF unit root test has the underlying null hypothesis that the variables are non-stationary at a certain significance level. The returns are obtained by taking the log difference of spot and future price. The stationary series are shown in Fig. 1(b), 2(b) and 3(b).

Table 1: Stationarity test of NIFTY50 index.

Null Hypothesis: SPOTPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=35)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.261192	0.6497
Test critical values:	1% level	-3.431055	
	5% level	-2.861736	
	10% level	-2.566916	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: FUTUREPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 30 (Automatic - based on SIC, maxlag=35)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.321740	0.6217
Test critical values:	1% level	-3.431055	
	5% level	-2.861736	
	10% level	-2.566916	

*MacKinnon (1996) one-sided p-values.

Table 2: Stationarity test of BANKNIFTY index.

Null Hypothesis: SPOTPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.078047	0.7266
Test critical values:	1% level	-3.431656	
	5% level	-2.862002	
	10% level	-2.567059	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: FUTUREPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 24 (Automatic - based on SIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.057494	0.7344
Test critical values:	1% level	-3.431660	
	5% level	-2.862004	
	10% level	-2.567060	

*MacKinnon (1996) one-sided p-values.

Table 3: Stationarity test of NIFTYIT index.

Null Hypothesis: SPOTPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=29)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.434705	0.9009
Test critical values:	1% level	-3.431844	
	5% level	-2.862085	
	10% level	-2.567104	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: FUTUREPRICESERIES has a unit root
Exogenous: Constant
Lag Length: 3 (Automatic - based on SIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.401251	0.9067
Test critical values:	1% level	-3.431839	
	5% level	-2.862083	
	10% level	-2.567103	

*MacKinnon (1996) one-sided p-values.

Descriptive or summary statistics of log difference of spot and futures prices of the indices is given in Table 4 and shows some important features of the data. The series has higher kurtosis called leptokurtosis indicates the existence of fatter tail and higher peak value around the mean than the normal distribution. In Table 4, the value of skewness indicates that all price levels have a longer tail on the left-hand side.

Table 4: Summary statistics for spot and future price of NIFTY 50, BANKNIFTY and NIFTYIT.

	NIFTY50		BANKNIFTY		NIFTYIT	
	Spot returns	Futures returns	Spot returns	Futures returns	Spot returns	Futures returns
Mean	0.00834	0.00845	0.01092	0.01093	0.00884	0.00886
Median	0.01885	0.01866	0.00978	0.00762	0.01179	0.01338
Standard Deviation	0.0605	0.06191	0.08237	0.08325	0.0675	0.06767
Kurtosis	3.99344	3.54815	4.20782	4.12542	4.14708	4.29127
Skewness	-0.85057	-0.77476	-0.20685	-0.22262	-1.0265	-1.06344
Minimum	-0.27033	-0.26776	-0.24001	-0.23966	-0.32242	-0.32674
Maximum	0.18146	0.18809	0.26485	0.26222	0.15917	0.15732
Sum sq.dev	0.43192	0.45221	0.80055	0.81779	0.53757	0.5404
Count	119	119	119	119	119	119
Jarque-Bera	84.74522	67.27746	8.08191	7.26292	96.75454	103.7208
Probability	0.00000	0.00000	0.01758	0.02648	0.00000	0.00000

The Jarque-Bera (JB) test statistics for detecting normality of data shows the rejection ofnull hypothesis that the financial time series taken in this research is normal. This is due to the presence of extreme values or high heteroscedasticity in the sample.

ARCH or heteroscedasticity test is performed in EVIEWS software using ARCH -LM test (Table 5-7). Hedging horizon is chosen as monthly.

Table 5: NIFTY50 ARCH test.

Heteroskedasticity Test: ARCH

F-statistic	0.152887	Prob. F(1,116)	0.6965
Obs*R-squared	0.155318	Prob. Chi-Square(1)	0.6935

Table 6: BANKNIFTY ARCH test.

Heteroskedasticity Test: ARCH

F-statistic	8.964235	Prob. F(1,116)	0.0034
Obs*R-squared	8.464660	Prob. Chi-Square(1)	0.0036

Table 7: NIFTYIT ARCH test.

Heteroskedasticity Test: ARCH

F-statistic	1.362472	Prob. F(1,116)	0.2455
Obs*R-squared	1.369873	Prob. Chi-Square(1)	0.2418

The models discussed in section III (B) are then applied to the return series and the parameters are estimated using the maximum likelihood estimation technique

(Table 8-10). Hedge ratio and hedging effectiveness are estimated using Eqns. 1 and 2 (Table 11). The corresponding hedged portfolios are thus formed and optimization is performed using FMOLP technique discussed in section III (C). The parameters for optimization are return, risk, Sharpe ratio, Treynor ratio, and CV. The values of these parameters are estimated using the data of the hedged portfolios.

Results and discussion. The first step in performing a research with financial data is to check the statistical properties of the data under consideration. Fig. 1(a) - 3(a) shows the spot and future price series of the indices taken. The result of stationarity test or the ADF unit root test (Table 1-3) shows that the spot and future price series are non-stationary.

Table 8: Parameter estimation of the three GARCH models for the index NIFTY50. The values in parenthesis denote the corresponding standard error.

Parameter	SGARCH	EGARCH	GJR-GARCH
Constant (ω1)	0.00006 (0.00010)	-6.22042 (2.65878)	0.00006 (0.00009)
ω2	0.00006 (0.00010)	-5.93732 (2.75136)	0.00006 (0.00010)
ω3	0.00025 (0.00041)	0 0	0 0
ω4	0 0	—	—
ARCH term(λ1)	0.14804 (0.08620)	0.45167 (0.16408)	0.06901 (0.10239)
λ2	0.14216 (0.08583)	0.43094 (0.16941)	0.06467 (0.10212)
λ3	0.14491 (0.08594)	0.13829 (0.01428)	0.13829 (0.01428)
4	0.10693 (0.05927)	—	—
GARCH term α1	0.83782 (0.09371)	-0.09687 (0.46362)	0.83901 (0.09302)
α 2	0.84408 (0.09337)	-0.05643 (0.48441)	0.83867 (0.09452)
α3	0.84115 (0.09352)	0.861712 (0.01515)	0.861712 (0.01515)
α4	0.83316 (0.05676)	—	—
Leverage term	—	-0.21594 (0.07936)	0.13303 (0.08043)
	—	-0.20912 (0.08443)	0.14632 (0.08901)

They are made stationary by taking log difference of spot and future price series (Fig. 1(b)-3(b)). Summary statistics of spot and future returns of the indices (Table 4) show excess kurtosis. This shows the existence of fat tails and high peakedness in the return distribution. Also, negative skewness shows that all price levels have an asymmetric tail extending towards the left-hand side. The result of Jarque-Bera (JB) test statistics is used to check the normality of data. The result of the test rejects the null hypothesis that the data under study is normal. ARCH–LM test is applied to test the heteroscedasticity in the data.

The result (Table 5-7) shows that the null hypothesis of having no ARCH effects in the data is rejected. A considerable amount of heteroscedasticity is found in the data. In this research, maximum likelihood estimation is used to estimate the parameters of the proposed models in MATLAB software and the results are listed in Table 8-10.

Table 9: Parameter estimation of the three GARCH models for the index BANKNIFTY. The values in parenthesis denote the corresponding standard error.

Parameter	SGARCH	EGARCH	GJR-GARCH
Constant (ω1)	0.00036 (0.00048)	-0.50114 (0.32714)	0.00042 (0.00038)
ω2	0.00031 (0.00046)	-0.51188 (0.33268)	0.00041 (0.00040)
ω3	0.00134 (0.00190)	0.00000 (0.00000)	0.00001 (0.00000)
ω4	0.00000 (0.00000)	—	—
ARCH term (λ1)	0.09929 (0.05368)	0.20056 (0.07478)	—
λ 2	0.08718 (0.05012)	0.19606 (0.07516)	—
λ3	0.09338 (0.05197)	0.15228 (0.03735)	0.15228 (0.03735)
λ4	0.13158 (0.06393)	—	—
GARCH term (α1)	0.8405 (0.10961)	0.90312 (0.06324)	0.81217 (0.08558)
α2	0.8606 (0.10425)	0.90065 (0.06461)	0.82125 (0.08662)
α3	0.84992 (0.10724)	0.78141 (0.03911)	0.78141 (0.03911)
α4	0.79895 (0.05584)	—	—
Leverage term	—	-0.22299 (0.08122)	(0.23886) (0.13098)
	—	-0.22518 (0.07914)	0.22163 (0.12712)

Hedging is performed with the help of GARCH models viz. SGARCH, EGARCH and GJR-GARCH model (Table 11). The strategy with minimum hedge ratio and maximum hedging effectiveness reduces the portfolio variance to a great extent.

In this regard, the results of SGARCH model are more consistent. Hedging only reduces the loss of spot portfolio. It does not guarantee profit or return. In this research, a multiobjective mean-CVaR portfolio optimization model is used. Along with risk and return, some other criteria like Sharpe ratio, Treynor ratio and CV are also adopted which helps an investor in the selection of assets in a portfolio. The hedged portfolios are formed using GARCH models discussed in section III. Table 12-14 shows the input data for the optimization of three portfolios formed using GARCH models.

Table 10: Parameter estimation of the three GARCH models for the index NIFTYIT. The values in parenthesis denote the corresponding standard error.

Parameter	SGARCH	EGARCH	GJR-GARCH
Constant (ω1)	0.00036 (0.00044)	-0.08556 (0.00718)	0.00022 (0.00025)
ω2	0.00033 (0.00044)	-0.06666 (0.0050)	0.00021 (0.00025)
ω3	0.00138 (0.00177)	0.00001 (0.00000)	0.00001 (0.00000)
ω4	0.00000 (0.00000)	—	—
ARCH term (λ1)	0.151932 (0.12761)	-0.16206 (0.07177)	—
λ2	0.14663 (0.12649)	(-0.15751) (0.05449)	—
λ3	0.14944 (0.12723)	0.31391 (0.18749)	—
λ4	0.19254 (0.13408)	—	—
GARCH term (α1)	0.77283 (0.20389)	0.98511 (0.00086)	0.82366 (0.15942)
α2	0.78344 (0.20276)	0.98783 (0.00033)	0.82875 (0.16190)
α3	0.77799 (0.20346)	0.68609 (0.14135)	0.68609 (0.14135)
α4	0.66028 (0.13172)	—	—
Leverage term	—	-0.19310 (0.04372)	0.23414 (0.10781)
—	—	-0.15482 (0.05221)	0.22740 (0.10912)

Table 11: Result of the hedge ratio and hedging effectiveness.

Parameters	NIFTY50	BANKNIFTY	NIFTYIT
	SGARCH		
Hedge ratio	0.97811	0.98900	0.99277
Hedging effectiveness	98%	98%	98%
	EGARCH		
Hedge ratio	1.08893	1.16180	1.58428
Hedging effectiveness	97%	95%	62%
	GJR-GARCH		
Hedge ratio	1.02170	1.09797	1.10641
Hedging effectiveness	98%	97%	97%

Table 12: Input data for SGARCH.

Index	Return	Risk	Sharpe ratio	Treynor ratio	CV
NIFTY50	0.00008	0.00521	3.50186	0.00852	32.56078
BANKNIFTY	0.00012	0.00578	2.93928	0.00806	23.74189
NIFTYIT	0.00005	0.00564	3.20681	0.00841	52.97835

Table 13: Input data for EGARCH.

Index	Return	Risk	Sharpe ratio	Treynor ratio	CV
NIFTY50	-0.00086	0.01616	3.05188	0.02250	-8.54932
BANKNIFTY	-0.00175	0.03190	2.61955	0.03801	-8.27799
NIFTYIT	-0.00519	0.07839	2.12984	0.08488	-7.68268

Table 14: Input data for GJR-GARCH.

Index	Return	Risk	Sharpe ratio	Treynor ratio	CV
NIFTY50	–0.00028	0.00799	2.78933	0.01012	-12.96358
BANKNIFTY	–0.00110	0.02080	2.50488	0.02417	-8.79509
NIFTYIT	–0.00099	0.01490	1.97959	0.01604	-8.21292

The result of optimization indicates that hedged portfolio constructed due to SGARCH hedging strategy gives the highest weight to BANKNIFTY while EGARCH and GJR-GARCH to NIFTY50 (Table 15). The results of Table 16 reveal that the hedged portfolio formed using SGARCH is much better than the other two portfolios. SGARCH hedging strategy provides the highest return with minimum risk, higher Sharpe and Treynor ratio and least positive CV. Also the result of Table 11 indicates that hedge ratio due to SGARCH model is minimum. This implies that this model can provide a maximum return with minimum investment as higher hedge ratios require higher investment. This information proves to be very useful to investors in their risk/return tradeoff. Table 16 shows the comparison of optimization between hedged portfolios formed due to hedging strategies of three GARCH models.

Table 15: Optimal weights for each index future.

Index	SGARCH	EGARCH	GJR-GARCH
NIFTY50	0.4220	0.5555	0.5555
BANKNIFTY	0.5555	0.4220	0.4220
NIFTYIT	0.0225	0.0225	0.0228

Table 16: Upper and lower bound for each objective.

Objective	ub (SGARCH)	lb (SGARCH)	ub (EGARCH)	lb (EGARCH)	ub (GJR-GARCH)	lb (GJR-GARCH)
Return	0.00010	0.00006	-0.00133	-0.00364	-0.00064	-0.00103
Risk	0.00571	0.00554	0.05737	0.02420	0.02419	0.01354
Sharpe ratio	3.18271	3.06484	2.84869	2.35725	2.65167	2.28961
Treynor ratio	0.008261	0.008215	0.03045	0.02370	0.01619	0.01618
Coefficient of variation	40.97857	28.12130	-7.95340	-8.41532	-11.1005	-11.9759

Table 17: Comparison of optimization between hedged portfolios.

Objective	SGARCH	EGARCH	GJR-GARCH
Return	0.00010	-0.001334	-0.00064
Risk	0.00554	0.02420	0.01354
Sharpe ratio	3.18271	2.84869	2.65167
Treynor ratio	0.00826	0.03045	0.01619
cv	28.12130	-8.41532	-11.10005

Comparative analysis. Comparing the three hedging strategies (Table 17), we observed that the SGARCH model performed better than the other two models. It generates a lower hedge ratio and higher hedging effectiveness. This implies that this strategy is very much effective than the other two strategies. It will remove maximum risk from spot portfolio with minimum investment. After optimizing the hedged portfolios, the portfolio formed using the SGARCH model generates higher returns and lower risk with a higher Sharpe ratio. This information will benefit the financial analysts in the selection of proper hedging strategy.

V. CONCLUSION

This research applies a Fuzzy Multiobjective Linear Programming (FMOLP) technique to optimize a hedged portfolio of stock index futures. The approach used in this research has the following components: hedging, CVaR, and FMOLP. Hedging has been done before optimization to minimize the unexpected price fluctuations of spot portfolio. The parameters of hedging are the hedge ratio and hedging effectiveness. Optimization of the hedged portfolio is performed by maximizing the objective of return, Sharpe and Treynor ratio while minimizing the objective of risk and coefficient of variation.

These parameters are helpful to investors and traders in choosing various assets in a portfolio. As the return and risk are uncertain, the FMOLP optimization technique is employed which converts a multiobjective optimization problem into a single objective using a “weighted adaptive approach”.

In this research, an equally weighted or 1/N-portfolio approach is used for estimating optimal weights. This optimization technique has the property that it allows choices in solution.

This implies that if the portfolio is not in favor of an investor, he can readjust the weights of the objective function according to his will to get better results. On comparing the portfolio performance of the proposed GARCH models, it is found that SGARCH reduces the risk of a portfolio to a certain extent as compared to the other two models. Also, the return generated from SGARCH hedging strategy is more than that of the other two models with a high Sharpe and Treynor ratio. This information is of great help to investors in making proper investment decisions in a futures market.

VI. FUTURE SCOPE

Hedging provides security to the investment and also helpsin reducing the losses borne by the investor due to unexpected fluctuations arises in the market. This study is an attempt to assess the power of hedging strategies using index futures. The study aims at providing an insight into the operation of hedging strategies. The study describes the strategies to select the right hedging techniques based on the requirements of the investors. If an investor wants to protect his investment for a longer time horizon, then this study helps him to select a better strategy as per his requirement.

Conflict of Interest. The authors of this research declare that they have no conflicts of interest.

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